Topology....

(Topological spaces :-)

Topology :- let x be a non empty set $x \neq \phi$ and let τ be collection of subsets of x, is called topology of x if

$[T_1]$: $\phi \in \tau$, $X \in \tau$

$[T_2]$: If $G_1 \in \tau$ and $G_2 \in \tau$ then $G_1 \cap G_2 \in \tau$

$[T_3]$: If union of any arbitary number of sets from τ is also in τ , $G_1 \cup G_2 \in \tau$

Then topology form and (x, τ) is called Topological space.

eg:- let $x = \{a, b, c\}$ and $\tau = \{\phi (a, b) (b)\ x\}$ show that it forms topology

$[T_1]$: $\phi \in \tau$ and $x \in \tau$ (satisfied)

$[T_2]$: let $G_1 = \{a, b\}$, $G_2 = \{b\}$ Now $G_1 \cap G_2 = \{b\} \in \tau$

$[T_3]$: Now $G_1 \cup G_2 = \{a, b\} \in \tau$

Hence all condition satisfied and (x, τ) is called topological space.

Indiscrete topology :- The topology consisting of only two elements namely ϕ and x is called Indiscrete topology of x and denoted by I.

Discrete topology :- The topology consisting of all subset of x is called discrete topology of x, and denoted by D

Note :- I is the smallest topology and D is the largest topology.

Adherent Point :- A point $x \in X$ is called an adherent point (or contact point) of A, iff every nbd of x contains atleast one point of A. i.e $x \in X$ is called adherent point of A iff $\exists$ nbd N of x. s.t $N \cap A \neq \phi$. The set of all adherent points of A is denoted by $adh(A)$ (5)

Isolated Point :- let (X, τ) be a topological space and $A \subset X$. A point $x \in A$ is called isolated point of A if x is not a limit point of A. In other words $\exists$ nbd G of x s.t $(G - \{x\}) \cap A = \phi$ or $G \cap A = \{x\}$. If every point of a set A is an isolated point of A then the set A is called isolated set

Dense Set :- let (X, τ) be a topological space and $A \subset X$

- A is dense in itself if $A \subset D(A)$
- A is said to be dense or every where dense if $\bar{A} = X$
- A is said to be dense in a set $B \subset X$ if $B \subset \bar{A}$
- A is said to be some where dense if $(\bar{A})^\circ \neq \phi$ i.e closure of A contains some open sets
- A is said to be no where dense if it is not some where dense. i.e A is said to be no where dense or non dense set in X if, $(\bar{A})^\circ = \phi$

X is said to be separable if $\exists$ $A \subset X$ s.t A is countable and $\bar{A} = X$.

eg. If $\tau = \{\phi, \{a\}, \{a,b\}, \{a, c, d\}, \{a, b, e\}, \{a, b, c, d\}, X\}$ be a topology on $X = \{a, b, c, d, e\}$ then,

i) Point out τ open subset of X.

τ open subsets of X are the elements of τ namely
ϕ $\{a\}$ $\{a,b\}$ $\{a,c,d\}$ $\{a,b,e\}$ $\{a,b,c,d\}$ X.

ii) Point out τ closed subset of X.

we know that $G \in \tau \Leftrightarrow G' = X - G$ is τ closed.

τ closed set are, ϕ, $\{a\}'$ $\{a b\}'$ $\{a, c d\}'$ $\{a b e\}'$ $\{a b c d\}'$ X'

that is. X, $\{b c d e\}$ $\{c d e\}$ $\{b e\}$ $\{c d\}$ $\{c\}$ ϕ